February 28, 2023

Demand Learning, Customer Capital, and Exporter Dynamics*

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The dynamics of exporters are extremely rich, with complicated patterns of entry, growth and exit. We develop a general equilibrium trade model in which these dynamics arise endogenously as a result of a customer accumulation friction and imperfect information about demand that can be resolved through learning. We show that this model does a good job of quantitatively replicating exporter dynamics from the data and provides new insights into the effects of trade liberalization and protectionism. Customer and information frictions account for nearly half of a standard estimate of iceberg costs, despite these frictions being symmetric for domestic and foreign firms. The effects of changes in trade costs are much more convex than in a simpler model that ignores the dynamics of firms. Relatively open economies gain more from trade liberalization than in the simpler model, while more closed economies lose less. For the US, the increase in GDP from trade liberalization is amplified by approximately a factor of two while losses from protectionism are compressed by half.

JEL codes: D83, D84, E22, F19

1 INTRODUCTION

Recent empirical research has documented a rich array of facts about the dynamics of exporters. These facts relate to their size when they start exporting, their growth dynamics over time, the dynamics of their prices

*For comments and suggestions, thank you to Doireann Fitzgerald, Luca Opromolla, Tom Sargent, Tatsuro Senga, Laura Veldkamp, Mike Waugh and seminar/conference participants at the Bank of Portugal, NYU, E1 Macro, Midwest Macro, CESifo Global Economy Area Conference, Annual Meeting of the Portuguese Economic Journal, ETSG, ESCB Research Cluster 2 Annual Workshop and the 2nd International Trade Dynamics Workshop. Thanks also to Piotr Denderski, Alberto Felettigh and Daniel Xu for their discussions of the paper.

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and the evolution of their survival rates, amongst other things.[1] This evidence raises the question of whether accounting for these dynamics matters for the aggregate effects of trade liberalization and protectionism. Once we account for these dynamics is a shift back towards protectionism more or less costly than previously thought? Are the potential gains from further liberalization as promising as previously predicted?

To address these questions at the aggregate level a general equilibrium model that incorporates these dynamics is needed. This paper develops such a model with firm dynamics driven by a process for customer accumulation and imperfect information about demand that firms can resolve through learning. A crucial feature of the model is that these ingredients generate firm dynamics as a result of endogenous decisions rather than exogenous shocks. This matters because it means that the dynamics of firms can respond to changes in the environment. This allows the model to capture the interaction between changes in trade costs and firm dynamics, making it well suited to counterfactual experiments.

In order to evaluate the importance of exporter dynamics for the aggregate effects of trade we calibrate the model to the data, and also calibrate a simplified version of the model that omits the new dynamics. This allows us to compare the effects of changes in trade costs in the two models and evaluate how accounting for the dynamics of firms changes things. We show that the full model does a good job of replicating exporter dynamics from the data. The first main result is that the iceberg trade estimate is almost half as large in the full model as the simple model, despite the full model having no additional trade-specific frictions. This indicates that a good share of what are typically estimated as "trade costs" may not be trade-specific frictions at all. The second main result is that accounting for exporting dynamics substantially changes the effects of changes in trade costs. The effect is not as simple as the economy becoming more or less sensitive to changes in these costs. It actually depends on the level of openness of the economy. If the economy is relatively open then firm dynamics make the economy *more* sensitive to changes in trade costs, while if the economy is relatively closed then it is *less* sensitive. The differences can be quantitatively large. For the US, accounting for exporter dynamics can amplify the increase in GDP from lower trade costs by a factor of two, and compress the losses from higher trade costs by half.

The model takes a standard Melitz (2003) model with stochastic productivity as its foundation and makes two adjustments in order to generate realistic firm dynamics. First it is assumed that firms face uncertainty about demand in each market that they sell in, and resolve this uncertainty by learning from their sales. Second, firms need to invest in marketing in order to acquire customers and the cost of this is convex—some customers are relatively easy to attract, but others are harder. These two features of the model interact because of the dual role of customer acquisition: an additional customer both increases potential sales and provides another person for a firm to learn about demand from, so that learning is faster. The model is deliberately designed to nest versions of Melitz (2003) so that it is easy to compare the effects of changes in trade costs in our model with the effects in previous models that have been used.

[1] See, for example, Ruhl and Willis (2017) and Fitzgerald, Haller, and Yedid-Levi (2017).

Consistent with empirical evidence, the model generates exporter dynamics with the following features on average: the initial quantity that a firm exports is positively correlated with the duration of its export spell; the quantity that a firm exports grows gradually over time; the probability that a firm exits an export market decreases with the length of the export spell; and, conditional on marginal cost, the price of an exporter is flat over time. The model generates these dynamics through the following mechanisms. Imperfect information about demand is key for the initial quantities of exporters being correlated with the lengths of their export spells. Many firms will try exporting to discover how high their foreign demand is. Low sales in the first period of exporting are a signal of low demand and low profitability, resulting in short export spells on average; and vice versa. Exporters take time to grow for two reasons. When a firm starts exporting it is uncertain about how much demand there will be and therefore holds back on investing in acquiring customers. As this uncertainty is resolved the firm learns more precisely how much it should invest in customer acquisition and, if appropriate, ramps up its investment. Also, because the cost of acquiring customers is convex within each period, firms invest gradually. Quantitatively, the effect of the customer friction is far more important for the gradual growth of firms.

There are also two reasons why the exporter exit rate declines over the course of an export spell. When a cohort of firms starts exporting, they have poor information about their demand. Many quickly learn that demand for their product is low and exit. Therefore there is selection on demand within a cohort over time and this drives the exit rate down. Customer acquisition plays a role because on average firms that have exported for longer have more customers. A larger customer base increases the value of exporting and therefore decreases the probability that a firm will exit a foreign market. The learning mechanism is quantitatively far more important for this moment. Finally our model yields the standard pricing formula from a CES demand structure with price being a constant markup over marginal cost. Therefore, conditional on marginal costs prices are constant over time, consistent with the data.

In order to assess how accounting for these dynamics alters the aggregate effects of changes in trade costs, we perform the following exercise. We start by setting up a second model that is identical to the full model except that customer capital accumulation and demand learning are removed. Instead the standard assumptions are used: firms can costlessly access all consumers; and there are no demand shocks so firms have perfect information about their demand. This is a Melitz (2003) model with stochastic productivity. With these two models in hand the experiment is to calibrate them both to the data and then compare the effects of increases and decreases in variable trade (iceberg) costs between them. Identical calibration moments are used for the two models, with some additional moments added to determine the richer dynamics of the full model. The data used for the calibration is primarily for US manufacturing establishments. Having calibrated these models to the data it is then possible to change the variable trade costs in the two models and compare the results.

The first main result arises from the estimates of iceberg costs for the two models. The full model produces an estimate that is only 57% as large as the estimate from the simple model, indicating that information and customer frictions can account for nearly half of the iceberg costs that are typically unmodeled. This provides insight in what exactly some of the frictions are that result in firms selling a lot less abroad than at home. Also, and perhaps more interestingly, these frictions are not trade specific and so would not typically be classed as trade costs. In the model firms face exactly the same information friction and customer acquisition costs in their home and foreign markets. Nevertheless, these costs act as a greater impediment to selling abroad than at home. The reason for this is that the customer and information frictions matter the most for firms when they first sell in a market and need to learn about their demand and acquire customers. Due to the fixed costs of exporting, firms don't operate for as long in the foreign market as the domestic market, on average, so the information and customer frictions matter more for exporters.

The second main result is that in the full model that accounts for exporter dynamics, the gains from lower trade costs are *greater* than in the full model, while the losses from higher trade costs are *smaller.* Note that this result is not as simple as the additional ingredients in the model making the economy more or less sensitive to changes in trade costs. The *sign* of the effect actually depends on how open the economy is to begin with. The effects are sizable. When the model is calibrated to US data the gains from trade liberalization increase approximately by a factor of two, while the losses from protectionism decrease by a half. One implication of this result is that if a relatively closed economy is gradually opening up to trade, then the gains from liberalization are a lot more back-loaded than a simpler model would suggested. This is a potential explanation for why sometimes the gains from partial trade liberalizations are not as large as people might have expected.

We perform a number of exercises to understand the mechanisms underlying this result. We produce a sharp decomposition of the relevance of the customer and information frictions for the aggregate effects. Despite being necessary to generate some of the micro-level facts about exporter dynamics, imperfect information about demand is virtually irrelevant for understanding the aggregate effects of trade. This is a very useful result since it shows that there are certain micro-level dynamic moments that can safely be ignored for the purposes of assessing the aggregate effects of trade.

We also assess what role the endogenous choice of customers plays, compared to an alternative in which firms accumulate customers at an exogenous rate over time. Without endogenous customer accumulation the main result holds qualitatively. The effect of endogenizing the customer choice is to amplify the gains/losses from trade liberalization/protectionism. When firms can adjust their investment in customers they invest more when trade costs fall and less when trade costs rise, generating these effects.

Literature review This paper is most closely related to three papers that consider how the dynamics of firms matter for the gains from trade. Atkeson and Burstein (2010) consider this question in an environment in which firms can grow over time through innovation. We study a different force driving the dynamics of firms and focus more on the ability of the model to qualitatively and quantitatively replicate the dynamics of exporters in the data. Alessandria and Choi (2014) and Alessandria, Choi, and Ruhl (2018) also study the relationship between firm dynamics and the effects of changes in trade policy, although they consider firm dynamics driven by supply side shocks (productivity and trade cost shocks) while we assess the role of demand side frictions.

A second related strand of literature studies the importance of customer capital and demand learning for exporter dynamics. Fitzgerald, Haller, and Yedid-Levi (2017) and Eaton, Eslava, Jinkins, Krizan, and Tybout (2014) both provide empirical evidence on the dynamics of exporters and argue that customer capital accumulation and learning about demand are important for generating the firm dynamics that we see in the export data. These papers develop partial equilibrium models and evaluate their ability to explain exporter dynamics. We contribute to this line of research by embedding these forces in a quantitative general equilibrium framework that allows us to evaluate how they impact the aggregate effects of trade. Ruhl and Willis (2017) and Berman, Rebeyrol, and Vicard (2015) also present evidence on the dynamics of exporters. The former argues that these dynamics can be explained by gradual demand accumulation and a mechanism that causes some firms who will be bad at exporting to try it (such as demand learning), while the latter argues in favor of demand learning. Arkolakis (2010) models customers in the context of exporting, but abstracts from their dynamic accumulation over time, which is central to the present paper.

There is a broader literature studying dynamics in international trade models that our research also relates to. Several papers consider theories of exporter dynamics based sunk costs (Impullitti, Irarrazabal, and Opromolla, 2013), financial frictions (Caggese and Cuñat, 2013; Gross and Verani, 2013; Kohn, Leibovici, and Szkup, 2016) and capital adjustment costs (Rho and Rodrigue, 2016; Liu, 2018). While these papers differ in various respects, none of them study how the dynamics of firms matters for the effect of changes in trade costs on GDP. Caggese and Cuñat (2013), Kohn, Leibovici, and Szkup (2016), Rho and Rodrigue (2016) and Liu (2018) use partial equilibrium models that are not suitable for addressing this question. Impullitti, Irarrazabal, and Opromolla (2013) and Gross and Verani (2013) have general equilibrium models, but are focused on understanding the dynamics of firms rather than how these dynamics matter for aggregate effects. Arkolakis (2016) is another paper with trade and firm dynamics, however its focus is on integrating these features into a growth framework and all firm dynamics are exogenous. A slightly different strand of the literature studies the dynamics of aggregate trade while abstracting from the lifecycle of individual firms that we focus on. These papers incorporate physical capital into trade models and aggregate dynamics are driven by the gradual adjustment of the capital stock. Within this class there are papers studying both long run dynamics (Bache, 2012; Anderson, Larch, and Yotov, 2015; Alvarez, 2017; Ravikumar, Santacreu, and Sposi, 2018) and cyclical fluctuations (Fattal Jaef and Lopez, 2014; Eaton, Kortum, Neiman, and Romalis, 2016).

The learning mechanism in this paper is related to that in a few other papers. The shared ingredients are that firms face idiosyncratic demand shocks with an unknown distribution and use equilibrium prices as signals by which they update their beliefs about the unknown distribution. The present paper differs by connecting learning to customer acquisition—and thereby giving firms control over their learning—and we also study a different application. Arkolakis, Papageorgiou, and Timoshenko (2015) show that in their learning environment firm growth decreases with age, holding firm size constant; and decreases with size, holding firm age constant. Timoshenko (2015) applies learning to a model where each firm produces multiple products and optimally chooses to switch the bundle of products they export in response to changes in beliefs. Bastos, Timoshenko, and Dias (2016) use data on Portuguese firms and a model with learning to study the quality choice of inputs by exporters.

Finally, this paper is related to previous research on customer capital in a closed economy context, such as Gourio and Rudanko (2014b), Gourio and Rudanko (2014a) and Perla (2016). Our paper differs from these by focusing on the relevance of customer capital for exporter dynamics and by connecting customer accumulation to demand learning.

2 MODEL

The model is an extension of a two country version of Melitz (2003). There are two modifications. The first modification is to drop the assumption that consumers know about all goods. Instead it is assumed that when a firm is born no conumers knows about it and that firms must invest over time to acquire customers. Second, firms do not know the demand for their products and must learn over time.

2.1 *Environment*

There are two countries, Home and Foreign. Each country has a unit measure of consumers who each supply one unit of labor inelastically and consume. We will just describe the environment for Home as the setup is symmetric for Foreign. Throughout the paper for any variable ι we let ι denote the value in Home and ι^* denote the Foreign value.

Each country produces a continuum of differentiated goods. The set of goods sold in Home in period t is denoted Ω_t. Some of these goods will be produced in Home and some in Foreign. Each consumer can only consume a good if she is aware of it. Let the set of goods that consumer i is aware of when consumption occurs in period t be denoted by $\Omega_{it} \subseteq \Omega_t$. This consumer values a CES aggregate of these goods:

$$C_{it} = \left(\int_{\Omega_{it}} \xi_{it}(\omega)^{1-\rho} q_{it}(\omega)^{\rho} \right)^{1/\rho}, \tag{1}$$

where ω indexes goods varieties, $q_{it}(\omega)$ is the quantity of the type ω good that is consumed by agent i, $\xi_{it}(\omega)$ is a preference shock for agent i for variety ω at time t and $\rho < 1$ is a parameter controlling the elasticity of substitution between goods. The preference shock for variety ω for agent i at time t is drawn from an exponential distribution with parameter $\lambda(\omega) > 0$. These draws are independent across consumers and over time. Note that the demand parameter is indexed by ω so demand is heterogeneous across firms. The reason for including preference shocks is so that firms can be uncertain about their demand and learn it over time. Consumers get income from working, they own all of the firms in the home country and receive their profits (each consumer has an equal share of each firm), and also receive tariff revenue which is collected by the government and given to them in a lump sum. The details of how tariffs are collected are explained below. We abstract from savings so the consumers consume all of their income each period. The discount rate of all consumers is $\theta \in (0,1)$.

Each good is produced by a single firm and each firm has only one good. At the start of each period firms can be created by employing γ_e units of labor per new firm. When a firm is created it draws a productivity level $z \in Z \equiv \left\{z^1, z^2, \ldots, z^{N_z}\right\}$ from the distribution $G(z)$. The firm also draws parameters for its preference shocks distributions in Home and Foreign, $\lambda(\omega) > 0$ and $\lambda^*(\omega) > 0$, which are assumed to be independent draws from $\Gamma(\alpha_\lambda, \beta_\lambda)$ and $\Gamma(\alpha^*_\lambda, \beta^*_\lambda)$.[2] A firm does not know the values of either of these draws initially. All it knows are the distributions from which the parameters are drawn. A new firm can start producing in the period in which it is created.[3] Firms produce using a linear production technology with labor as the only input. If firms choose to export then they must pay a fixed cost of γ_x (denoted in units of labor) each period. They are also subject to two variable costs: an "iceberg" cost τ and a tariff ψ.[4] Tariffs are collected by the government and given in lump sum to the household. The government has no other role in the model. After its first period the productivity of a firm will evolve according to a Markov process $G(z_{t+1}|z_t)$.

A firm can only sell its variety to consumers who are aware of it. When a firm is born no consumers know about it in either country and it must invest in attracting them. At the start of each period firms choose how much to invest. In order to attract $I \in \mathbb{N}$ new customers in Home a firm must hire $\Phi(I)$ units of Home labor, and the cost of $I^* \in \mathbb{N}$ new consumers in Foreign is $\Phi(I^*)$, denoted in units of Foreign labor.[5] Φ is assumed to satisfy the following properties: $\Phi(1) = 0$, $\Phi' > 0$ and $\Phi'' > 0$. We assume the first property so that all firms have at least one customer to sell to at all times. The motivation for an increasing and convex cost is that some consumers are harder to reach or attract than others, so the marginal cost of customers increases. An increasing and covex cost also follows the literature on customer capital (Klepper, 2002; Arkolakis, 2010; Dinlersoz and Yorukoglu, 2012; Drozd and Nosal, 2012; Gourio and Rudanko, 2014b). Let $N_t(\omega)$ denote the number of agents that know about variety ω in Home at the start of period t. The number of consumers that the firm can sell to in period t is therefore $N_t(\omega) + I_t(\omega)$, which we will call the Home *customer capital* of this firm. Between periods t and $t+1$ the firm will lose each customer with probability δ_N (and these events are independent across customers and over time). For the foreign country the number of consumers at the start of period t is $N^*_t(\omega)$ and the probability of losing each consumer each period is also δ_N. If a firm ever stops exporting then its customer capital in the export market will return to zero.

Firms will learn about the distribution of their demand shocks from the customers that they sell to. Each period a firm will set its price in Home and Foreign and observe the quantities it sells in each market. These quantities will provide information to the firm.

[2]The pdf for the $\Gamma(\alpha_\lambda, \beta_\lambda)$ distribution is $f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ for $x \in (0, \infty)$. The assumption that preference shocks for a firm are drawn from an exponential distribution with its parameter drawn from a gamma distribution are made for analytical tractability. Our results are not dependent on these particular distributions, as is discussed later in the paper.

[3]There is no fixed cost of operating in a firm's local market, so all firms that are created will produce and sell in that market.

[4]These variable costs are implemented so that if a firms ships $q(1+\tau)(1+\psi)$ units of a good then it will be able to sell q units.

[5]We define the natural numbers $\mathbb{N}$ to include 0.

At the end of each period each firm dies with probability δ.

We will now solve the problems of a consumer and a firm in Home. The problems for consumers and firms in Foreign are analogous. We'll then describe the equilibrium of the model.

2.2 Consumer's problem

The problem for consumer i is to maximize consumption subject to her budget constraint:

$$\max_{\{q_{it}(\omega)\}_{\omega\in\Omega_{it}}} \quad C_{it} \tag{2}$$

$$\text{s.t.} \quad \int_{\Omega_{it}} p_t(\omega) q_{it}(\omega) d\omega \leq Y_t, \tag{3}$$

where $p_t(\omega)$ is the price of a variety ω in Home and Y_t is nominal aggregate income in Home.[6] We will provide an equation defining Y_t after discussing the firm's problem. This consumer's problem generates the following demand functions for domestic and foreign varieties:

$$q_{it}(\omega) = \frac{Y_t}{P_t} \left(\frac{P_t}{p_t(\omega)} \right)^{\frac{1}{1-\rho}} \xi_{it}(\omega) \tag{4}$$

where P_t is the Home price index defined as

$$P_t := \left(\int_{\Omega_{it}} p_t(\omega)^{\frac{\rho}{\rho-1}} \xi_{it}(\omega) d\omega \right)^{\frac{\rho-1}{\rho}}.$$

Note that the i in this equation can be any consumer in the economy. This is because there is a continuum of firms whose customers are drawn randomly from the population, so although each consumer will be aware of a different set of firms, the joint distribution of prices and preference shocks over the varieties that each consumer knows about will be the same. This distribution will also be the same as the distribution of these characteristics over all products available in the market. Therefore the price index can be written as:

$$P_t = \left(\frac{M_{it}}{M_t} \int_{\Omega_t} p_t(\omega)^{\frac{\rho}{\rho-1}} \xi_{it}(\omega) d\omega \right)^{\frac{\rho-1}{\rho}}. \tag{5}$$

where $M_{it} := |\Omega_{it}|$ is the mass of goods that consumer i knows about (which has the same value for all consumers), and $M_t := |\Omega_t|$ is the mass of goods sold in Home.

2.3 Firm's problem

A firm has five decisions to make: how much to invest in customer capital in Home, what price to set for Home, whether to export and, if it exports, how much to invest in customer capital and what price to set for Foreign. We will first describe the production problems for the two countries, then the consumer capital investment problems and finally the export problem.

[6] Note that all agents in Home have the same income because they own equal shares in all Home firms, supply the same amount of labor and earn the same wage. Since there is a unit measure of agents this means that $Y_{it} = Y_t$.

Production problems Let the set of consumers who know about variety ω when production occurs in period t be denoted $\mathcal{I}_{\omega t}$, so $\mathcal{I}_{\omega t} := \{i : \omega \in \Omega_{it}\}$. The demand that the firm producing this variety faces is

$$q_t(\omega) = \sum_{i \in \mathcal{I}_{\omega t}} q_{it}(\omega) = \frac{Y_t}{P_t} \left(\frac{P_t}{p_t(\omega)} \right)^{\frac{1}{1-\rho}} \xi_t(\omega) \tag{6}$$

where $\xi_t(\omega) := \sum_{i \in \mathcal{I}_{\omega t}} \xi_{it}(\omega) \sim \Gamma(N_t(\omega) + I_t(\omega), \lambda(\omega))$.[7]

For the Home market each firm sets its price and then hires enough labor to produce the quantity that is demanded at that price. The firm's production problem is

$$\begin{aligned} \max_{p_t(\omega)} \quad & \mathbb{E}_{\omega t}[q_t(\omega) p_t(\omega) - w_t l_{Ht}(\omega) | p_t(\omega)], \\ \text{s.t.} \quad & l_{Ht}(\omega) = \frac{q_t(\omega)}{z_t(\omega)} \\ & q_t(\omega) \text{ determined by equation (6)} \end{aligned}$$

where $l_{Ht}(\omega)$ is the quantity of labor the the firm uses to produce goods for the Home market and $\mathbb{E}_{\omega t}$ denotes the expectation taken with the information of firm ω at the time of production in period t. Making use of demand function (4), this generates an optimal price of

$$p_t(\omega) = \frac{1}{\rho} \frac{w_t}{z_t(\omega)}. \tag{7}$$

Given this price, the firm's output is given by equation (4) and the amount of labor it will need to hire to meet demand is

$$l_{Ht}(\omega) = \frac{Y_t}{P_t} \left(\frac{\rho P_t}{w_t} \right)^{\frac{1}{1-\rho}} z_t(\omega)^{\frac{\rho}{1-\rho}} \xi_t(\omega). \tag{8}$$

The firm's realized profit from the domestic market is

$$\pi_{Ht}(\omega) = (1-\rho) Y_t \left(\frac{\rho z_t(\omega) P_t}{w_t} \right)^{\frac{\rho}{1-\rho}} \xi_t(\omega). \tag{9}$$

Conditional on exporting a firm must also choose what price to set in Foreign. The problem is analogous to the production problem for Home and the firm's optimal price is

$$p_t^*(\omega) = \frac{1}{\rho} \frac{w_t(1+\tau)(1+\psi)}{z_t(\omega)}. \tag{10}$$

The firm's labor demand and profit are:

$$l_{Ft}(\omega) = \frac{Y_t^*}{P_t^*} \left(\frac{\rho P_t^*}{w_t} \right)^{\frac{1}{1-\rho}} \left(\frac{z_t(\omega)}{(1+\tau)(1+\psi)} \right)^{\frac{\rho}{1-\rho}} \xi_t^*(\omega), \tag{11}$$

$$\pi_{Ft}(\omega) = (1-\rho) Y_t^* \left(\frac{\rho z_t(\omega) P_t^*}{w_t(1+\tau)(1+\psi)} \right)^{\frac{\rho}{1-\rho}} \xi_t^*(\omega). \tag{12}$$

[7] $\xi_t(\omega)$ has this distribution because it is the sum of i.i.d. random variables from an exponential distribution.

Belief evolution As we will show shortly, the evolution of a firm's beliefs about its demand shocks in Home and Foreign is important for its decisions about how much to invest in customer capital in each country. For firm ω the demand shocks for Home are drawn from an exponential distribution with parameter $\lambda(\omega)$. When a firm is born its beliefs about this parameter have the distribution $\Gamma(\alpha_\lambda, \beta_\lambda)$.

A firm will obtain new information about the distribution of its demand shocks in Home each period that it produces. Let firm ω's beliefs about $\lambda(\omega)$ at the start of period t be denoted $\Gamma(\alpha_t(\omega), \beta_t(\omega))$ (this is the *prior* distribution). After trade between firms and consumers occurs in period t firm ω knows its realized quantity $q_t(\omega)$ and can back out the sum of its demand shocks $\xi_t(\omega)$ using equation (6) because it can observe aggregate variable P_t and Y_t. This information is a signal about $\lambda(\omega)$ and the firm updates its beliefs about this parameter using Bayesian updating. This produces posterior beliefs that have a distribution $\Gamma(\alpha_{t+1}(\omega), \beta_{t+1}(\omega))$, where

$$\alpha_{t+1}(\omega) = \alpha_t(\omega) + N_t(\omega) + I_t(\omega), \tag{13}$$

$$\beta_{t+1}(\omega) = \beta_t(\omega) + \xi_t(\omega). \tag{14}$$

For a firm that exports we denote its beliefs about $\lambda^*(\omega)$ at the start of period t by $\Gamma(\alpha_t^*(\omega), \beta_t^*(\omega))$ and the updating rules are the same as equations (13) and (14) with $N_t^*(\omega)$, $I_t^*(\omega)$ and $\xi_t^*(\omega)$ replacing their counterparts for the Home country.

Customer capital and export problems Each period, prior to production, a firm must choose how much to invest in customer capital in each country. First consider the Home country problem. The number of customers that firm ω enters period t with in Home is $N_t(\omega)$. The choice of how much to invest in customer capital is a dynamic problem since a firm retains each customer with probability $1 - \delta_N$ from one period to the next, so a firm's investment in customer capital today will affect its profits in future periods. To streamline the notation we will omit the ω that indexes the firm in writing this problem and drop time subscripts. Instead for any variable A we denote A_t by A and A_{t+1} by A'. Let $V(z, N, \alpha, \beta)$ be the value of the Home country operations of a Home firm at the start of a period. This value is

$$V(z, N, \alpha, \beta) = \max\nolimits_{I \in \mathbb{N}} \mathbb{E}[\pi_H(z, \xi) | N + I, \alpha, \beta] - \Phi(I)w + (1-\delta)\theta \mathbb{E}[V'(z', N', \alpha', \beta') | z, \alpha, \beta, N + I] \tag{15}$$

$$\begin{aligned}
\text{s.t.} \quad z' &\sim G(z'|z), \\
N' &\sim B(N + I, 1 - \delta_N), \\
\alpha' &= \alpha + N + I, \\
\beta' &= \beta + \xi, \\
\xi &\sim \Gamma(N + I, \lambda)
\end{aligned}$$

where B denotes the binomial distribution.

The firm faces a similar problem for determining whether to export and sell its goods to Foreign and how many new customers to obtain in Foreign. Let $V^*(z, N^*, \alpha^*, \beta^*)$ be the value of of Foreign operations for a

Home firm entering a period with productivity z, N^* current Foreign consumers, and beliefs α^* and β^* about λ^*. The value is written recursively as

$$\begin{aligned} V^*(z,N^*,\alpha^*,\beta^*) = \max_{I^*\in\mathbb{N},x\in\{0,1\}} & x\left(\mathbb{E}[\pi_F(z,\xi^*)|N^*+I^*,\alpha^*,\beta^*] - \Phi(I^*)w^* - \gamma_x w\right) \\ & + (1-\delta)\theta\mathbb{E}[V^{*'}(z',N^{*\prime},\alpha^{*\prime},\beta^{*\prime})|z,\alpha^*,\beta^*,N^*+I^*] \end{aligned} \tag{16}$$

$$\text{s.t.} \quad z' \sim G(z'|z),$$

$$N^{*\prime} \begin{cases} \sim B(N^*+I^*, 1-\delta_N) & \text{if } x=1, \\ = 0 & \text{if } x = 0, \end{cases}$$

$$\alpha^{*\prime} = \begin{cases} \alpha^* + N^* + I^* & \text{if } x=1, \\ \alpha^* & \text{if } x=0, \end{cases}$$

$$\beta^{*\prime} = \begin{cases} \beta^* + \xi^* & \text{if } x=1, \\ \beta^* & \text{if } x=0, \end{cases}$$

$$\xi^* \sim \Gamma(N^*+I^*, \lambda).$$

Now that we have discussed the firm's problem we can provide expressions for tariff revenue and aggregate income in the home country. To help with these expressions let $\Omega_{Ht} \subseteq \Omega_t$ be the set of varieties produced by Home firms. Tariff revenue is

$$T_t = \psi \int_{\Omega_t \setminus \Omega_{Ht}} p_t(\omega) q_t(\omega) d\omega \tag{17}$$

and aggregate income is

$$Y_t := w_t + \int_{\Omega_H} \pi_{Ht}(\omega) - \Phi(I(\omega))w_t + x_t(\omega)\Big(\pi_{Ft}(\omega) - \Phi(I^*(\omega))w_t^* - \gamma_x w_t\Big) d\omega - M_{et}\gamma_e w_t + T_t. \tag{18}$$

2.4 *Firm entry*

At the start of each period a firm will be created if its expected value exceeds the cost of creating it. The free entry condition is therefore:

$$\mathbb{E}[V(z,0,\alpha_\lambda,\beta_\lambda) + V^*(z,0,\alpha^*_\lambda,\beta^*_\lambda)] = \gamma_e w. \tag{19}$$

Let the mass of new firms created at the start of period t be denoted M_{et}.

2.5 *Market clearing conditions*

The markets that need to clear in Home are the labor market and the market for each variety. The labor market clearing condition is

$$M_{et}\gamma_e + \int_{\Omega_H} l_{Ht}(\omega) + \Phi(I(\omega)) + x_t(\omega)(l_{Ft}(\omega) + \gamma_x) d\omega + \int_{\Omega \setminus \Omega_H} \Phi(I(\omega)) d\omega = 1. \tag{20}$$

The market clearing conditions for varieties that are produced in Home are

$$q_t(\omega) = z_t(\omega) l_{Ht}(\omega) \ \ \forall\, \omega \in \Omega_H \tag{21}$$

and the market clearing conditions for varieties exported from Foreign to Home are

$$q_t(\omega) = \frac{z_t^*(\omega) l_{Ht}^*(\omega)}{(1+\tau)(1+\psi)} \ \ \forall\, \omega \in \Omega \setminus \Omega_H. \tag{22}$$

An additional condition that the equilibrium must satisfy is that the total mass of connections to consumers that firms have equals to the total mass of connections to firms that consumers have:

$$\int_{\Omega_t} (N_t(\omega) + I_t(\omega)) d\omega = M_{it}. \tag{23}$$

2.6 Firm distribution

Let the state space, S, over $(z, N, \alpha, \beta, \lambda, N^*, \alpha^*, \beta^*, \lambda^*)$ be the cartesian product $Z \times \mathbb{N} \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{N} \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$,[8] and let the σ-algebra Σ_S be defined as $P(Z) \otimes P(\mathbb{N}) \otimes B_{\mathbb{R}_+} \otimes B_{\mathbb{R}_+} \otimes B_{\mathbb{R}_+} \otimes P(\mathbb{N}) \otimes B_{\mathbb{R}_+} \otimes B_{\mathbb{R}_+} \otimes B_{\mathbb{R}_+}$ where $B_{\mathbb{R}_+}$ is the Borel σ-algebra on $\mathbb{R}_+$ and $P(X)$ denotes the power set of a set X. Let $\mathcal{S} = (\mathcal{Z} \times \mathcal{N} \times \mathcal{A} \times \mathcal{B} \times \mathcal{L} \times \mathcal{N}^* \times \mathcal{A}^* \times \mathcal{B}^* \times \mathcal{L}^*)$ be the typical subset of Σ_S. For any element $\mathcal{S} \in \Sigma_S$ let $F_t(\mathcal{S})$ be the time t measure of firms in set $\mathcal{S}$.

Let $Q : S \times \Sigma_S \to [0,1]$ be the transition function for the distribution of incumbent firms. Specifically,

$$Q\left((z, N, \alpha, \beta, \lambda, N^*, \alpha^*, \beta^*, \lambda^*), (\mathcal{Z} \times \mathcal{N} \times \mathcal{A} \times \mathcal{B} \times \mathcal{L} \times \mathcal{N}^* \times \mathcal{A}^* \times \mathcal{B}^* \times \mathcal{L}^*)\right)$$

is the conditional probability that a firm with current state $(z, N, \alpha, \beta, \lambda, N^*, \alpha^*, \beta^*, \lambda^*)$ transitions into the set $(\mathcal{Z} \times \mathcal{N} \times \mathcal{A} \times \mathcal{B} \times \mathcal{L} \times \mathcal{N}^* \times \mathcal{A}^* \times \mathcal{B}^* \times \mathcal{L}^*)$ next period. Note that the dynamic decisions of the firm—their export decision and their customer capital investment choices—are functions of their state variables. Specifically the functions are $x(z, N^*, \alpha^*, \beta^*)$, $I(z, N, \alpha, \beta)$ and $I^*(z, N^*, \alpha^*, \beta^*)$. To make the notation manageable we omit the arguments of these functions when writing down the transition function.

$$\begin{aligned}
Q\big((z, N, \alpha, \beta, &\lambda, N^*, \alpha^*, \beta^*, \lambda^*), (\mathcal{Z} \times \mathcal{N} \times \mathcal{A} \times \mathcal{B} \times \mathcal{L} \times \mathcal{N}^* \times \mathcal{A}^* \times \mathcal{B}^* \times \mathcal{L}^*)\big) \\
&= (1-\delta)(1-x)\Big\{\mathbb{1}\left\{\alpha + N + I \in \mathcal{A}, \lambda \in \mathcal{L}, 0 \in \mathcal{N}^*, \alpha^* \in \mathcal{A}^*, \beta^* \in \mathcal{B}^*, \lambda^* \in \mathcal{L}^*\right\} \\
&\quad \times \sum_{z' \in \mathcal{Z}} G(z'|z) \sum_{N' \in \mathcal{N}} f_B(N'; N+I, 1-\delta_N) \int_{\beta' \in \mathcal{B}} f_\Gamma(\beta' - \beta; N+I, \lambda) d\beta' \Big\} \\
&\quad + (1-\delta) x \mathbb{1}\Big\{ \{\alpha + N + I \in \mathcal{A}, \lambda \in \mathcal{L}, \alpha^* + N^* + I^* \in \mathcal{A}^*, \lambda^* \in \mathcal{L}^*\} \\
&\quad \times \sum_{z' \in \mathcal{Z}} G(z'|z) \sum_{N' \in \mathcal{N}} f_B(N'; N+I, 1-\delta_N) \sum_{N^{*\prime} \in \mathcal{N}^{*\prime}} f_B(N^{*\prime}; N^* + I^*, 1-\delta_N) \\
&\quad \times \int_{\beta' \in \mathcal{B}} f_\Gamma(\beta' - \beta; N+I, \lambda) d\beta' \int_{\beta^{*\prime} \in \mathcal{B}^*} f_\Gamma(\beta^{*\prime} - \beta^*; N^* + I^*, \lambda^*) d\beta^{*\prime} \Big\},
\end{aligned} \tag{24}$$

[8] We define $\mathbb{R}_+ := (0, \infty)$.

where $f_B(n; N, p)$ is the p.m.f. for the Binomial distribution and $f_\Gamma(x; \alpha, \beta)$ is the p.d.f. for the Gamma distribution.[9]

To understand (24), consider how a firm at state $(z, N, \alpha, \beta, \lambda, N^*, \alpha^*, \beta^*, \lambda^*)$ transitions to a new state in the next period. There are three possible cases:

1. With probability $1 - \delta$ the firm survives to the next period. If this firm chooses not to export ($x = 0$), then the transition is shown in the first two lines of equation (24). $\lambda, N^*, \alpha^*, \beta^*$ and λ^* don't change for this case and given the choices of the firm α evolves deterministically. z evolves according to $z' \sim G(z'|z)$, $N' \sim B(N + I, 1 - \delta_N)$ and $\beta' = \beta + \xi$ where $\xi \sim \Gamma(N + I, \lambda)$. To understand the evolution of β recall that $\xi_t(\omega) = \sum_{i \in \mathcal{I}_{\omega t}} \xi_{it}(\omega)$ so in the recursive notation ξ is a sum of $N + I$ i.i.d. random variables from an exponential distribution with parameter λ. The sum of these random variables has distribution $\Gamma(N + I, \lambda)$. Note that this law of motion depends on the true distribution of the demand shocks rather than the distribution of shocks that the firm believes that it faces.

2. If the surviving firm chooses to export ($x = 1$) then its transition is described in lines 3 to 5 of the right hand side of equation (24). For this case λ and λ^* do not change, and α and α^* evolve deterministically given the firm's choices for customer capital investment in the two countries. Productivity evolves according to $G(z'|z)$, the number of customers in each country evolve according to Binomial distribution $B(N + I, 1 - \delta_N)$ and $B(N^* + I^*, 1 - \delta_N)$, and β and β^* evolve according to Gamma distributions $\Gamma(N + I, \lambda)$ and $\Gamma(N^* + I^*, \lambda^*)$.

3. Finally, with probability δ the firm dies and leaves the economy.

We also need to consider how newly created firms are distributed. Let $Q_e : \Sigma_S \to [0, 1]$ be a p.d.f. such that $Q_e(\mathcal{S})$ is the probability that the state variables for a newly created firm are in the set $\mathcal{S}$. This p.d.f is defined as:

$$\begin{aligned} Q_e\big(\mathcal{Z} \times \mathcal{N} \times \mathcal{A} \times \mathcal{B} \times \mathcal{L} \times \mathcal{N}^* \times \mathcal{A}^* \times \mathcal{B}^* \times \mathcal{L}^*\big) \\ = \mathbb{1}\left\{0 \in \mathcal{N}, \alpha_\lambda \in \mathcal{A}, \beta_\lambda \in \mathcal{B}, 0 \in \mathcal{N}^*, \alpha_\lambda \in \mathcal{A}^*, \beta_\lambda \in \mathcal{B}^*\right\} \\ \times \sum_{z' \in \mathcal{Z}} G(z') \int_{\lambda' \in \mathcal{L}} f_\Gamma(\lambda'; \alpha_\lambda, \beta_\lambda) d\lambda' \int_{\lambda^{*\prime} \in \mathcal{L}^*} f_\Gamma(\lambda^{*\prime}; \alpha_\lambda, \beta_\lambda) d\lambda^{*\prime} \end{aligned} \tag{25}$$

To understand this note that all new firms start out with $N = N^* = 0$, $\alpha = \alpha^* = \alpha_\lambda$, and $\beta = \beta^* = \beta_\lambda$. The initial value of z is drawn from $G(z)$, and λ and λ^* are both drawn from $\Gamma(\alpha_\lambda, \beta_\lambda)$.

Let $F_t : S \to [0, 1]$ be the distribution of firms over states at time t . Then for each $\mathcal{S} \in \Sigma_S$, the distribution of states evolves according to:

$$F_t(\mathcal{S}) = \frac{M_{H,t-1} \int_{s \in S} Q(s, \mathcal{S}) dF_{t-1} + M_{e,t} Q_e(\mathcal{S})}{(1 - \delta) M_{H,t-1} + M_{e,t}}. \tag{26}$$

[9] The notation $\mathbb{1}\{a, \ldots, b\}$ represents an indicator function taking the value 1 if all of $a, \ldots, b$ are true and 0 otherwise.

where $M_{H,t-1}$ is the mass of Home firms at the end of period $t-1$. We will be focusing on an equilibrium of the model in which the mass of firms in each country is constant over time, so

$$M_H = (1-\delta)M_H + M_e. \tag{27}$$

For this case we define a distribution F to be *stationary* if for all $\mathcal{S} \in \Sigma_S$

$$F(\mathcal{S}) = \int_{s \in S} Q(s,\mathcal{S})dF + \frac{M_e}{M_H} Q_e(\mathcal{S}). \tag{28}$$

2.7 Equilibrium

We focus on the symmetric stationary equilibrium of the model. Symmetric in the sense that all aggregate variables and distributions are identical for the two countries and stationary in the sense that all aggregate variables and distributions are constant over time. Since the equilibrium is symmetric, its definition can be stated using variables and functions for one country only: they are all identical for the second country.

A *symmetric stationary competitive equilibrium* is a wage w, a price index P, aggregate income Y, a mass of firms M_H, a mass of entrants M_e, a mass of goods that each consumer knows about M_i, tariff revenue T, a demand function for each good that a consumer knows about $q(\cdot)$, pricing policies for firms $p(\cdot)$ and $p^*(\cdot)$, customer capital investment policies for firms $I(\cdot)$ and $I^*(\cdot)$, an export policy $x(\cdot)$ and a stationary distribution of firms over their idiosyncratic states $F(\mathcal{S})$, such that:

1. Consumer demand satisfies (4);
2. Pricing policies satisfy (7);
3. The customer capital investment policy for Home solves (15);
4. The export policy and the customer capital investment policy for Foreign satisfy (16);
5. The price index satisfies (5);
6. The mass of firms that each consumer knows about satisfies (23);
7. The mass of entrants satisfies (27);
8. The free entry condition (19) holds;
9. Tariff revenue satisfies (17);
10. Aggregate income satisfies (18);
11. Markets clear in accordance with (20), (21) and (22); and
12. The stationary distribution of firms over their idiosyncratic states satisfies (28).

A summary of the algorithm used for solving the equilibrium is in the appendix.

3 CALIBRATION

In this section we calibrate two versions of the model. First we calibrate the full model that's presented in Section 2. Then we calibrate a simplified version of the model in which we drop demand shocks and the friction to acquiring customers. We will call these models the *full* and *simplified* models respectively. We will use the simplified model in the results section in order to evaluate how much the dynamics of firms influence the effects of trade. We calibrate both versions of the model so that one period is one year and primarily use moments of the data for US manufacturing establishments. Where the relevant moments of the data for US manufacturing establishments are not available, we use other sources.

3.1 Calibration of full model

To perform numerical exercises we need to specify the functional forms for the distribution of initial productivities $G(z)$, the Markov process for productivity, $G(z'|z)$ and the cost function for acquiring customers $\Phi(I)$, and calibrate the model. For the Markov process we assume that:

$$\log(z') = \rho_z \log(z) + \sigma_z \varepsilon \tag{29}$$

where $\varepsilon \sim N(0,1)$. For the distribution of initial productivities we use the stationary distribution implied by this process. The cost function for acquiring customers is assumed to take the form $\Phi(I) = a(I-1)^\eta$, with $a > 0$ and $\eta > 1$.

The full model has 14 parameters, six of which are calibrated externally, two are normalized and the remainder are calibrated internally. We set the discount rate, θ, equal to 0.96 following Alessandria, Choi, and Ruhl (2018). We choose the value of ρ so that the elasticity of substitution between goods equals 4. For the death rate of firms, δ, we use the exit rate of manufacturing establishments between 1991 and 1992 in the Business Dynamics Statistics (BDS) dataset from the Census Bureau: 9.6%.[10] While there is uncertainty about the value of the customer depreciation rate, Gourio and Rudanko (2014b) provide a range of evidence on this and settle on a value of 0.15, which we also use. We set the autocorrelation of the productivity process to 0.9354, following Alessandria, Choi, and Ruhl (2018), and the tariff to 10% which is about the level of trade barriers in the US.

The two parameters that are normalized are γ_e, the entry cost, and α_λ, one of the parameters that determines the the distribution of productivity shock parameters (i.e. the distribution of λ). γ_e is chosen to normalize the mass of firms in the model to 1. α_λ is chosen to normalize the average value of demand shocks to 1 as well. The unconditional expected value of a demand shock is $E[\xi] = \beta_\lambda/(\alpha_\lambda - 1)$, so we set $\alpha_\lambda = \beta_\lambda + 1$.

This leaves six parameters of the model which we calibrate by simulated method of moments using six moments of the data. We choose the parameter values to minimize the sum of the squared percentage deviations

[10]This dataset provides summary statistics for practically the universe of establishments in the private non-agricultural sector of the economy on an annual basis.

of the moments in the model from their data counterparts. While the parameters are jointly determined by all moments, the heuristic relationships between the parameters and moments are as follows. We choose the fixed cost of exporting (γ_x) targeting the share of firms that export, which is 22.3% in the 1992 Census of Manufacturers (Alessandria, Choi, and Ruhl, 2018). We also target the ratio of foreign sales to total sales for all exporters from this data (13%: Alessandria, Choi, and Ruhl, 2018). We use this moment to determine the iceberg trade cost, τ. To pin down the standard deviation of the productivity process, σ_z, the coefficient of variation of the size of US manufacturing establishments, measured with the number of employees, is used. For the US this moment was between 4 and 5 from 1974 to 2006 (Henly and Sánchez, 2009), and we take the middle the this range (4.5) as the target.

To calibrate the second parameter of the distribution of demand shock parameters, β_λ, we use the exit rate of firms from the export market in their first year, which is 37% in the data (Ruhl and Willis, 2017).[11] The variance of the demand shock parameter is $V[\lambda] = \alpha_\lambda/\beta_\lambda^2$. The larger β_λ is, the smaller the variance of the demand shock parameters, which reduces the likelihood that a firm starts exporting, learns that its demand in the export market is very low and exits.

This leaves the two parameters of the cost function for acquiring customers to calibrate. For the level of the cost, which is determained by a, we target a ratio of marketing costs to GDP of 6.6%, following Arkolakis (2010).[12] In the model the ratio of marketing costs to GDP in Home is:

$$\frac{w \int_\Omega \Phi(I(\omega)) d\omega}{Y_t}$$

The second parameter of the cost function for acquiring customers, η, controls the convexity of this cost which affects how rapidly firms will acquire customers. For this parameter we target the share of employment at firms in their first year of production. For the data we use the share of employees in the manufacturing sector who work at plants aged less than 1 in 1991 (from the BDS), which is 2.6%. For the model we use the share of Home labor used by Home firms in their first year of existence. The labor used in Home by a firm of type ω in its first year (let this be year t) is:

$$l_{Ht}(\omega) + \Phi(I_t(\omega)) + x_t(\omega)(l_{Ft}(\omega) + \gamma_x).$$

This includes production labor, labor used to recruit customers, and labor used to cover the fixed cost of exporting.[13]

[11] This is the unconditional probability that a firm that is a new exporter in the data in year t is not exporting in year $t + 1$. This moment is also from the Columbian data and we're assuming the same dynamics for US manufacturing establishments.

[12] Arkolakis (2010) presents evidence that advertising expenses were 2–2.5% of GDP for 1982–2007 in the US, that this increases to 4–5% when you include other forms of marketing (brand sponsorship and public relations, sales promotion and interactive marketing) and is 7.7% if you include marketing events (e.g. trade shows, telephone sales, supporting product material and the cost of hiring outside marketing personnel) as well. He considers a broad definition of marketing and picks a value of 6.6%. We follow this broad definition since in our model marketing covers all of a firm's activities that contribute to attracting customers. While these figures for marketing costs are for all industries, we use them since we are not aware of direct evidence for the costs of marketing in manufacturing. We are assuming that the costs are the same for manufacturing as for the economy as a whole.

[13] We omit the entry cost from the employment at firms in their first year. This is because we think of the entry cost as covering a

The parameter values for the full model are presented in Table 1 and the values of the calibration moments in the model and data are presented in Table 2.

3.2 *Simplified model and calibration*

In order to assess how demand learning and customer capital change the effects of trade on the economy we will compare the results for the full model to results for a simplified model which omits these features. There are two changes to the full model. First, firms do not face demand shocks so $\xi_{it}(\omega) = 1 \ \forall \ \omega, i, t$. Second, firms don't have to invest in order to make customers aware of their product. Instead we revert to the usual assumption that all agents know about all goods in the economy. Full details of how these changes affect the equations of the model are presented in the appendix.

Since the purpose of the simplified model is to compare how the effects of trade liberalization differ when we ignore the dynamics of firms, we calibrate the model to match the same data as we use to calibrate the full model (dropping 3 the three moments that were used to calibrate the parameters controlling demand uncertainty and customer accumulation).

The simplified model has nine parameters to calibrate. We calibrate four of these externally using the same values from the literature as for the full model: the discount rate θ is 0.96, the CES parameter is $\rho = 0.75$, the firm death rate is $\delta = 0.096$ and the tariff is $\psi = 0.1$. The entry cost γ_e is normalized so that the mass of firms in each country is 1. We set the value of the autoregressive parameter of the productivity process (ρ_z) to equal the same value as we use for the full model.

This leaves three parameters that we calibrate using three moments of the data. The fixed export cost γ_x is chosen to target the share of firms that export (22.3%); the iceberg cost τ to target the foreign sales to total sales ratio for all exporters (13%); and the standard deviation of the productivity process, σ_z, to target the coefficient of variation of the size of establihsments (4.5), with size measured with the number of employees. The parameter values and calibration moments are presented in Tables 1 and 2, respectively.

From here the paper proceeds to discuss two areas of results. First we explain the dynamics of firms in the model and the forces driving these. Then the focus turns to assessing how accounting for these dynamics changes the aggregate effects of trade. This section includes a discussion of the key differences between the parameters of the simple and full models, and why these arise.

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wide range of costs that a firm can incur from years before it hires any employees (which is when a firm shows up in the data) until several years after this point. Also many of these costs will be in the form of services obtained from other businesses (e.g. lawyers, accounting services, IT services, marketing services). In the model the entry cost is a simplification of the entry process in the form of a single cost in units of labor incurred in the first year of production. Since this does not map closely to the data we omit it from the calculation of employment at first year firms.

Parameter	Description	Model value	
		Full	Simplified
θ	Discount rate	0.96	0.96
ρ	CES parameter	0.75	0.75
δ	Firm death rate	0.096	0.096
δ_N	Customer depreciation rate	0.15	–
γ_x	Fixed export cost	0.06	0.019
τ	Iceberg cost	0.63	1.11
ψ	Tariff	0.1	0.1
γ_e	Entry cost	1.83	2.00
(a, η)	Customer acquisition cost function	(0.41, 1.07)	–
$(\alpha_\lambda, \beta_\lambda)$	Distribution of λ and λ^*	(2.8, 1.75)	–
(ρ_z, σ_z)	Productivity process	(0.94, 0.11)	(0.94, 0.27)

Table 1: Parameter values

Moment	Model value		Data
	Full	Simplified	
Share of firms that export	18.6%	18.5%	22.3%
Foreign/total sales of exporters	9.3%	9.6%	13%
Coefficient of variation of firm size	4.1	4.5	4.5
1^{st} year exit rate of exporters	39.4%	–	37%
Marketing costs/GDP	6.3%	–	6.6%
Share of employment at firms in 1^{st} year	2.9%	–	2.6%

Table 2: Calibration moments

4 FIRM DYNAMICS

In this section we discuss the drivers of firm dynamics in the model. Specifically we cover: the forces driving the growth of firms; the process through which firms learn about their demand; the dynamics of prices; and the forces driving entry and exit from the export market. Since the focus is on a symmetric equilibrium in which the aggregate variables (P, Y, w) are equal in the two countries, the * superscripts will be omitted.

Firm growth The model has been set up to generate gradual growth of firms as they age, consistent with what is observed in the data. The gradual growth occurs in both the domestic and export markets. In this section the focus is on the growth of exporters, although the growth of firms domestically is qualitatively the same.

To start it is useful to make the potential sources of firm growth clear. Using equations (6) and (7) the quantity that a firm sells in the export market can be expressed as:

$$q_t^*(\omega) \propto z_t(\omega)^{\frac{1}{1-\rho}}(N_t^*(\omega) + I_t^*(\omega))\bar{\xi}_t^*(\omega)$$

where $\bar{\xi}_t^*(\omega)$ is the average value of demand shocks that firm ω receives in period t: $\bar{\xi}_t^*(\omega) \equiv \xi_t^*(\omega)/(N_t^*(\omega) + I_t^*(\omega))$. This equation shows that a firm's exports can grow through three channels: productivity growth, growth in the amount of customer capital that the firm has ($N^* + I^*$), and through changes in the average demand shock.

The contribution of each channel to the growth of firms over long export spells is quantified in Figure 1a. It shows the average growth of each channel over the first 15 years of exporting, for export spells of at least that length. For each variable the level in the first year is normalized to one. The figure shows that there is little change in productivity or the average demand shock. This should not be too surprising since new firms draw their productivity from the stationary distribution so there is no productivity growth amongst firms on average. Demand shocks are i.i.d. over time so on average there is no growth in these. A priori it is possible that successful exporters could turn out to be firms who happen to receive increasing string of productivity or demand shocks, but this is not the case. Instead, Figure 1a shows that firms are growing through the gradual accumulation of customers.

The rate of accumulation of customers depends on a firm's investment choice. Recall from the statement of the export problem in equation (16) that a firm's decision about how much to invest in customer capital depends on its productivity, the size of its existing stock of customers (N^*), and its beliefs about its distribution of demand shocks (α^* and β^*). Since the cost of acquiring customers and a firm's expected value from an additional customer are independent of the number of existing customers that a firm has N^*, the choice of investment in customers is independent of N^*.

To understand the role of beliefs and productivity in shaping investment we perform two experiments. We start by simulating the model restricting attention to export spells that last for at least 15 years to provide enough time to observe the growth of firms. The average path of investment over these spells is plotted with the blue line labelled *Full model* in Figure 1b. This line shows that on average firms get one customer in the first period, increase their investment in the second period, and then investment declines and flattens out at about 1.05 from the fourth year onwards. The two experiments are to recompute this average path of investment (i) holding z fixed and (ii) holding beliefs (α and β) fixed at their values from the first period in which each firm exports. The first experiment tells us about the effect of learning on investment because the effects of changes in productivity are turned off. The result of this experiment is plotted with the maroon line label *z fixed* in Figure 1b. This tells us that learning pushes investment up over the course of long export spells as these successful exporters are learning that their demand is higher than their initial expectation. The gap between the blue and maroon lines captures the effect of changes in productivity. On average firm productivity is higher in periods 2–7 of exporting than in the first period—which pushes investment up—and then decreases a little. This shows that on average these exporters experience positive productivity shocks in their early years of exporting, but mean reversion of the productivity process eventually catches up with them.

The results of the second experiment are presented with the green line in Figure 1b labelled (α, β) *fixed*. The line shows that if firms' beliefs don't improve from the first to second period of exporting then investment falls rather than increasing and some firms exit (we know that some firms exit because the average level of investment is less than one). This again demonstrates the importance of learning for investment. It also shows that some firms whose current beliefs and productivity are not good enough to export profitably, do so in order to learn. They are exporting because there is a chance that they will have high demand in the foreign market, but they exit if this turns out to not be the case.

To provide additional detail about the evolution of beliefs, Figure 2 quantifies the change in the expected value of a demand shock, $E[\xi_{it}]$, over the course of export spells of various lengths. For firms that export for only one period, $E[\xi_{it}]$ is normalized to one. The expectations for export spells of all other lengths are plotted relative to this (for clarity only export spells that last for an odd number of years are plotted). For example, take the yellow line which is for export spells that are 7 years long. In the first year of exporting these firms' demand expectations are about the same (on average) as those of firms who only export for one period. However, their expectations then increase so that in the fifth year of exporting they expect each demand shock to be about 50% higher. Note that this is an increase in the expected value of a single demand shock, so any increase in the number of customers affects demand above and beyond this. Looking at Figure 2 as a whole, it shows that firms that export for longer have demand expectations that increase more rapidly and reach higher levels. For firms exporting for at least 15 years, the expectation of a demand shock after 10 years is about 150% higher than for firms who only export for a single year.

Putting this all together, the process through which exporters grow can be summarized as follows. They start out with no customers and low beliefs about their demand (relative to firms who have exported for some years). Some firms even expect negative profits in their first period of exporting, but bear this cost for the chance of learning that they have high demand. After learning about their demand in the first period, some firms exit while others ramp up their investment in customers after getting a positive signal about their demand. Firms continue to learn as they export. For firms that export for many periods, on average their beliefs about their demand improve over time, which spurs further investment in customers.

To compare the growth of firms in our model to the data we replicate an empirical exercise from Fitzgerald, Haller, and Yedid-Levi (2017). Figure 2 in that paper shows how the quantity of exports of Irish firms evolves over the course of export spells, as a function of the export spell length. The authors control for firm level supply side factors in the exercise in order to focus on demand side forces. We replicate their analysis by generating simulated data from the model and running the following regression:

$$\log(q_t(\omega)) = \beta_0 + \beta_1 \log(z_t(\omega)) + \boldsymbol{\beta_2'}(\boldsymbol{a_t(\omega)} \otimes \boldsymbol{s_t(\omega)}) + \epsilon_t(\omega), \tag{30}$$

where $\boldsymbol{a_t(\omega)}$ is a vector of indicator variables for the number of years that firm ω has been exporting for in its current export spell as of period t (export tenure) and $\boldsymbol{s_t(\omega)}$ is a vector of indicator variables for the total length of the export spell that firm ω is currently in (spell length). So for example, if period t is the second year that a firm has been exporting and this export spell ultimately lasts for 6 years, then the variable in $\boldsymbol{a_t(\omega)}$

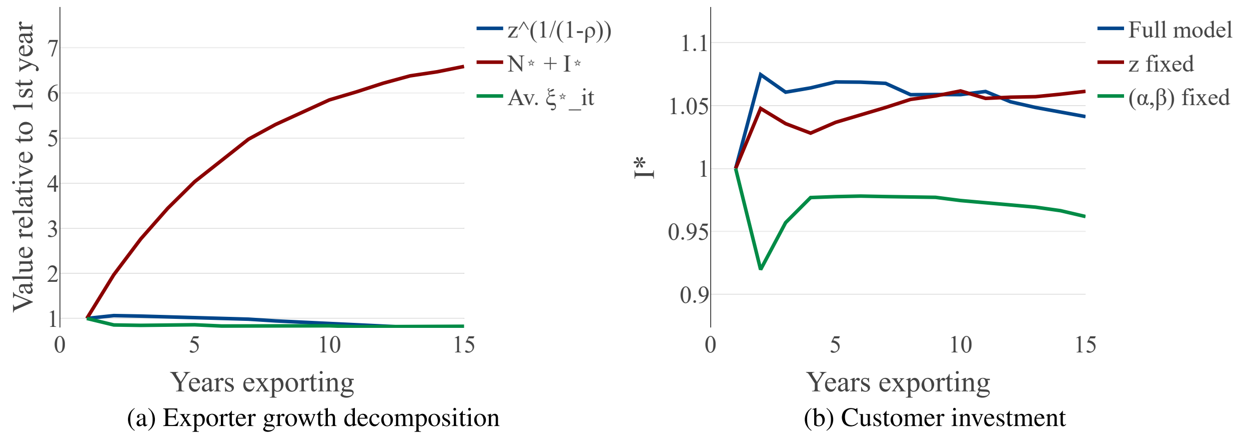


Figure 1: Forces determining the growth of exporters. Panel (a) is the average evolution of variables that determine the volume of exports for export spells of at least 15 periods. Panel (b) shows the evolution of I^* for export spells of at least 15 years, as well as counterfactual series in which productivity z is held fixed for each firm from the first export period onwards, and in which the beliefs of firms (α, β) are held fixed from the first export period.

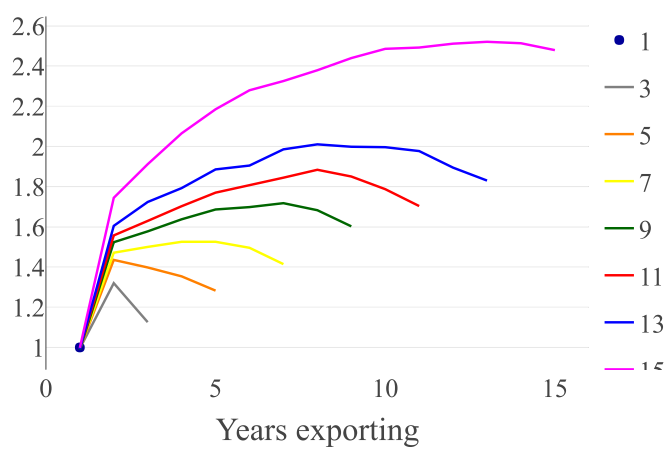
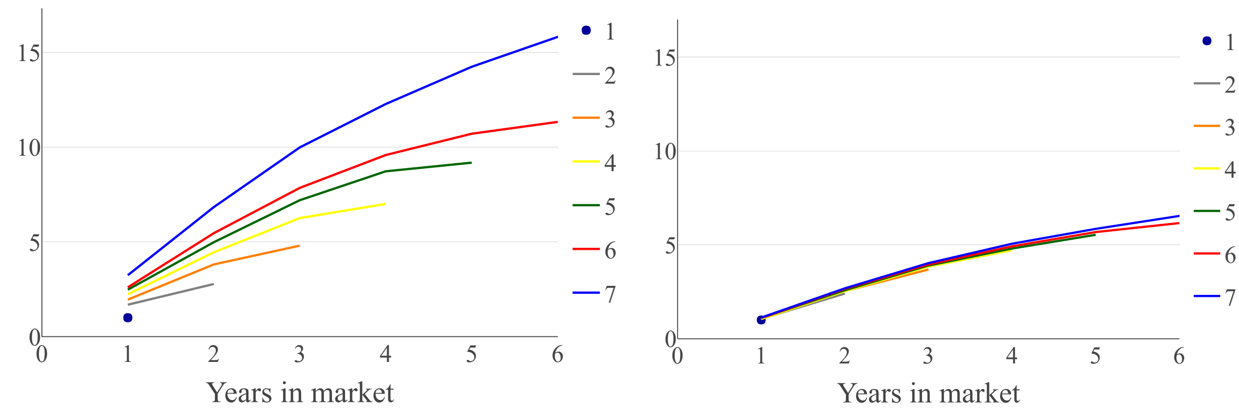


Figure 2: Evolution of beliefs by export spell length. This figure shows the average evolution of $E[\xi_{it}]$ for exporters, conditional on the length of the export spell. Export spells with an even number of years are omitted for clarity.



(a) Full model

(b) Perfect info. model

Figure 3: Export quantity by length of export spell. The figure was generated by simulating a large panel of firms and extracting the corresponding moments from the panel. Each line is for exports spells for the specified number of years. For 1 year long spells, the quantity is normalized to 1. All other quantities are given relative to this value. E.g. For export spells that last for 6 years (the red line), in the 6th year the average quantity exported is about 11 times as large as the average quantity exported by firms who only export for 1 year. The line labeled '7' is for export spells of 7 or more years.

corresponding to 2 years takes a value of 1 and the variable in $s_t(\omega)$ that corresponds to 6 years takes a value of 1. All other variables in $a_t(\omega)$ and $s_t(\omega)$ will equal to 0. We omit the indicator variable for export spells that last only one year so that this is the reference group

By taking the exponential of the coefficients in β_2 we can see how export quantities vary with export tenure and spell length. Figure 3a presents these results (for comparison with the data see Fitzgerald, Haller, and Yedid-Levi (2017), Figure 2). Each line plots how the quantity that a firm sells evolves, on average, over the course of an export spell of the specified length. The reference group is firms that export for only one year, so this quantity is normalized to 1 (the blue dot) and all other quantities are relative to this. The green line, for example, shows the evolution of quantity for firms that export for 5 years and then exit. In the first year they export about 2.5 times as much as firms who only export for one year, and then their quantity increases over time. In year 2 they sell about 5 times as much, in year 3 it is about 7 times as much and by year 5 about 9 times as much. The last line, labeled "7," is for firms who export for at least 7 years.

None of the features of this graph have been targeted in the calibration, so comparing the results to the data provides a test of whether the model generates reasonable exporter dynamics. Overall the model generates similar dynamics to what is in the data. Firms that export for longer start out smaller, firms grow gradually over time, firms that have longer export spells grow more quickly, and they also grow larger. Quantitatively the growth is also similar to in the data. For example, firms the export for at least 7 years in the model grow to be about 17 times larger than 1 year exporters, compared to 15 times in the data. The main difference between the model and data is that in the data exporters tend to shrink before exiting whereas in the model this does not happen.

In order to separate the roles of demand learning and customer capital in generating these exporter dynamics, we introduce a special case of the full model that isolates the role of customer capital. In this special case all firms know their demand distributions for Home and Foreign at birth, rather than having to learn about them over time.[14] We will refer to this model as the *perfect information model.* The evolution of quantities for exporters in this model are presented in Figure 3b. The main difference between this figure and the equivalent figure for the full model is that there is no correlation between the length of a firm's export spell and how much it exports in the first year. A firm that only exports for one year is, on average, the same size as other new exporters that survive in the Foreign market for 6 years. The reason for this is that with perfect information the selection of exporters happens pre-entry, rather than post-entry. In the full model, there are many firms that start exporting, discover that they have low demand and then exit. These types of firms account for most of the short export spells in the full model. In contrast, in the perfect information model these firms never start exporting. All the firms that do start exporting have a sufficiently high demand and productivity combination to justify this. After entry to the export market, firms will grow gradually over time as they accumulate customers, and quantitatively the growth rate is similar to what is observed for firms with long export spells (7 or more years) in the full model. Over time some firms will exit, but not because their demand is low. The reason for exit is that firms get poor productivity shocks or a death shock. The main message from this exercise is that customer capital is the main cause of gradual firm growth in the model, while learning about demand generates the positive correlation between initial exporter size and export spell duration in the data.

Learning mechanics Next consider the process through which firms learn about demand. Recall that the parameter $\lambda^*(\omega)$ determines the shape of the distribution from which firm ω's Foreign demand shocks are drawn, and all firms start with the rational belief that this parameter is distributed $\Gamma(\alpha_\lambda, \beta_\lambda)$. In order to decide whether to export and how much to invest in Foreign customer capital a firm must form beliefs about its demand shocks. Using standard properties of the gamma and exponential distributions a firm's expected demand shock in period t can be expressed as

$$E[\xi_{it}|\alpha_t^*, \beta_t^*] = \frac{\beta_t^*}{\alpha_t^* - 1}.$$

Equations (13) and (14) tell us how α^* and β^* evolve over time. β^* increases by the sum of a firms demand shocks each period, while α^* increases by the number of demand shocks received. This process has the property that if a firm receives demand shocks in period t that on average are greater than what it expected, then its expectation of its demand shock increases next period. That is, if

$$\xi_t^*/(N_t^* + I_t^*) > \frac{\beta^*}{\alpha_t^* - 1}$$

then $E[\xi_{i,t+1}^*|\alpha_{t+1}^*, \beta_{t+1}^*] > E[\xi_{it}^*|\alpha_t^*, \beta_t^*]$. So the beliefs of firms that reeive unexpectedly good demand shocks improve over time, while the beliefs of those with worse shocks than expected deteriorate.

[14]Technically, this means that firm ω know $\lambda(\omega)$ and $\lambda^*(\omega)$ at birth.

Price dynamics Recent empirical research has shown that while the quantities that firms export to a foreign market on average increase over the course of an export spell, as they do in our model, their prices are flat after controlling for marginal cost (Fitzgerald, Haller, and Yedid-Levi, 2017). For example, on average a firm exporting an aircraft part from Ireland to France charges the same price every year that it exports. It does not increase or decrease its price over time. In the model the price that a firm charges in the export market is given by equation (10). As is standard for models in which consumers consume a CES aggregate of differentiarted goods, the price is a constant markup over marginal cost, which is detemined by the wage and productivity. Therefore once changes in marginal cost are controlled for, the price of an exporter is constant over time, consistent with the data. In our model the only source of fluctuations in marginal cost is changes in productivity, since the wage is constant in the stationary equilibrium.

The way that the model reconciles customer capital accumulation with flat prices is to separate the pricing decision from the technology that determines the customer capital stock. Firms hire labor for marketing purposes in order to attract customers at the start of each period and then, given the number of customers that they have, they set prices for them. The price only affects demand on the intensive margin in the current period. This approach to the technology for customer accumulation is consistent with a number of existing papers in the literature (e.g. Arkolakis, 2010; Dinlersoz and Yorukoglu, 2012; Drozd and Nosal, 2012). An alternative to this technology would be to have prices play a role in customer capital accumulation, such as in the customer markets literature (e.g. Phelps and Winter, 1970; Bils, 1989; Klemperer, 1995; Ravn, Schmitt-Grohé, and Uribe, 2006; Gourio and Rudanko, 2014b). A common feature of models that take this approach is that firms charge low prices initially to attract customers and then increase their prices once relationships are formed. This approach is more difficult to reconcile with the fact that we see flat prices over the course of export spellls in the data.

Export entry and exit dynamics As well as the dynamics of firms within the export market, the model also has selection of exporters through entry into and exit from exporting. On the entry side, the value of a firm starting to export depends on its productivity z, and α^* and β^* which determine its beliefs about demand in the Foreign market. Firms that have never exported before have the same beliefs so they will export if their productivity exceeds a specific threshold. Firms that have exported before will have learned about their Foreign demand and have beliefs that differ from α_λ and β_λ and their productivity thresholds for exporting will be a function of these parameters.

Once a firm starts exporting it may exit because there is a fixed cost. The decision about whether to continue exporting or exit depends on the same three variables as the entry decision, as well as the amount of customer capital that a firm has, N^*. Figure 4 shows the role that these variables play in the selection of firms that leave the export market versus those who remain in it. This figure plots the values of z_{t+1}, N^*_{t+1} and $E[\xi_{it}|\alpha_{t+1}, \beta_{t+1}]$ for firms that decide to leave the export market at the start of period $t+1$ relative to their values for firms that decide to continue to export, as a function of the length of the export spell. For example, the points for 10 years of exporting show that firms that choose to leave the export market after 10 years have about 90% as much customer capital, about 90% as much productivity and 45% of the level of expected demand as firms who have exported for the same length of time, but choose to continue. So selection of exporters is occurring along all three margins, but the difference between those that exit and continue to export is largest in the dimension of demand.

Figure 5a shows the probability that a firm which has been exporting for x years continues exporting in the following period. In the first year of exporting firms have about a 62% probability of continuing to export in the next year. This probability gradually increases over the first 6 years of exporting, and then flattens out at a survival rate of 85%. At this point most of the exits are due to the exogenous death shock, and about 5 percentage points worth are due to endogenous selection. Figure 5b compares the survival rate to the data (compare the lines labelled “Full model” and “Data”). The survival rate after the first year of exporting is a calibration target, but the rest of the data points are free moments. The figure shows that the model generates an increase in the survival rate as export tenure increases, as we see in the data. Quantitatively the model generates 68% of the increase in the survival rate that is in the data (an increase of 19 percentage points, compared to 28 percentage points).

There are two mechanisms in the model that generate the upward slope in the survival rate. First, as firms export they learn about their demand. Firms with lower demand are more likely to exit (Figure 4) and those that continue to export tend to have beliefs about demand that improve over time (Figure 2). Thus there is selection on demand amongst exporters so that firms who have exported for longer tend to have higher demand and therefore a higher value of continuing to export. The second mechanism is that firms accumulate more customers the longer they export for (Figure 1a). Having more customers increases the value of exporting and pushes down the probability of exiting.

To disentangle the contributions of these mechanisms we return to the perfect information model introduced earlier in this section, in which firms know their demand distributions in Home and Foreign at birth. In this model there is no exit due to learning, so the increase in the survival rate with exporter age comes from more mature exporters having more customers. The green line in the figure, labeled “Perfect info”, shows that this mechanism has quantitatively small effects as the line only increases slightly with age. So most of the increase in the survival rate with age is coming from selection on demand.

5 AGGREGATE EFFECTS OF TRADE LIBERALIZATION AND PROTECTIONISM

Having explained the dynamics of firms in the model, we now consider how accounting for these dynamics matters for the aggregate effects of trade. In particular the focus will be on on how these dynamics alter the effects of changes in the iceberg cost.

In order to assess the importance of the dynamics of firms both the full model and the simple model that was laid out in Section 3 are used. Recall that the simple model is setup in exactly the same way as the full model,

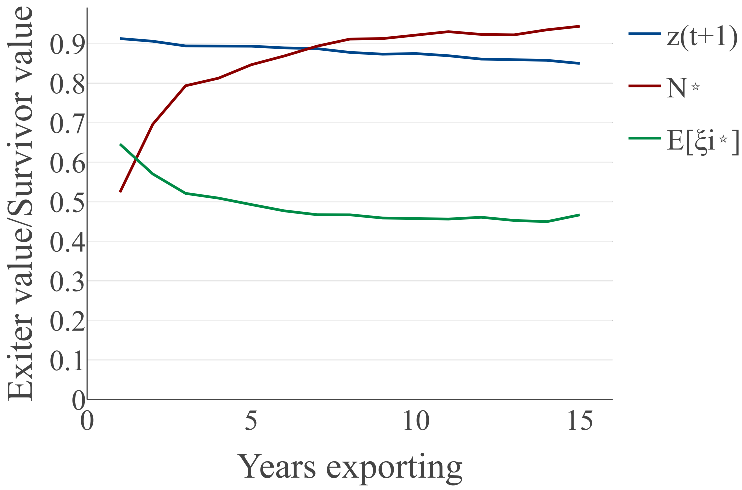
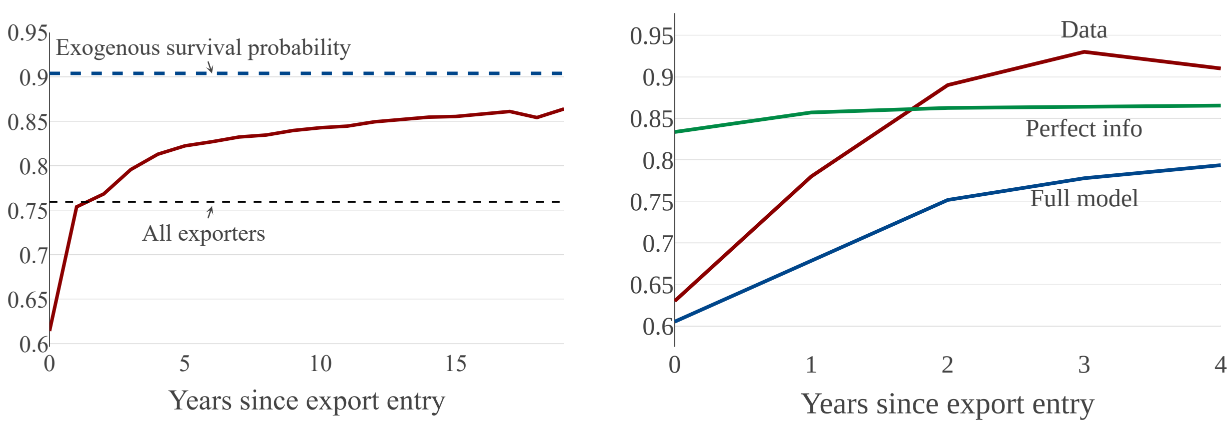


Figure 4: Selection of exporters. This figure shows the relative values of z_{t+1}, N^*_{t+1} and $E[\xi_{i,t+1}]$ for firms that stop and continue exporting as a function of the number of years that the firms have been exporting for. The sample excludes firms that stop exporting because they receive an exogenous death shock.



(a) Model

(b) Model and data

Figure 5: Exporter survival rates. Panel (a) presents the probability that a firm exports in the next period if it started exporting t periods ago. Panel (b) is the same moment presented alongside the data values from Ruhl and Willis (2017).

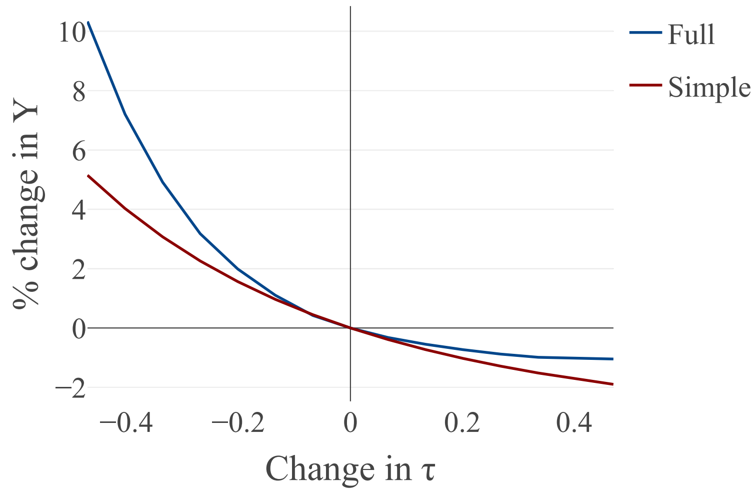


Figure 6: Effects of equal iceberg cost changes in full and simple models. This figure shows the percentage change in Y as a function of the change in τ for the simple and full models. All changes are relative to the baseline calibration for the relevant model.

except that we drop two of the features that cause individual firms to evolve over time—customer capital accumulation and demand learning. The experiments involve making changes to the iceberg cost in the two models and comparing the effects. The results will tell us whether accounting for the dynamics of individual firms matters for the aggregate effects of these changes in trade costs.

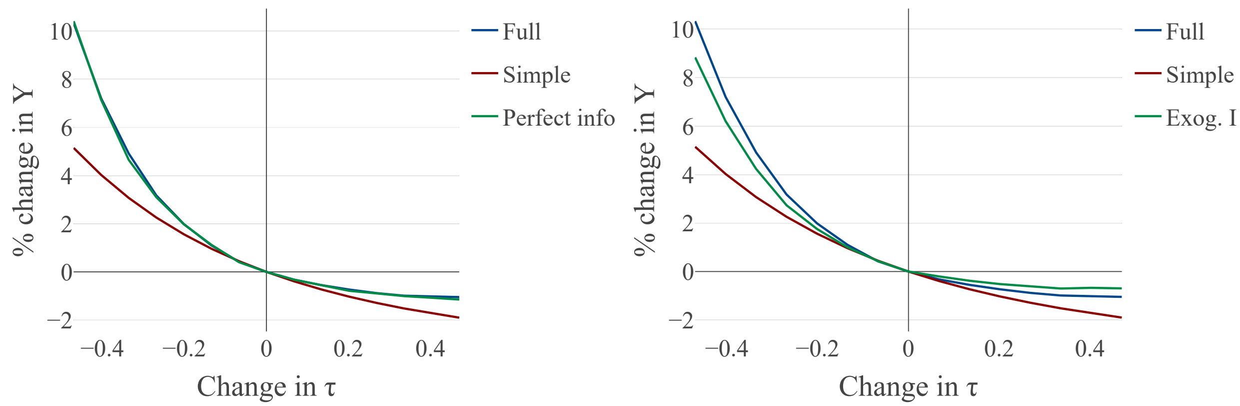
For now the experiments focus on how changes in the parameters affect the steady states of the models. In a future draft we will also account for the transition path of the dynamic economy between steady states.

To assess the effects of changes in the iceberg cost, we take the two calibrated models and increase and decrease τ by up to 0.45 points in both the models.[15] The main focus is on the effect on aggregate income Y. The headline results are presented in Figure 6. The horizontal axis is the change in τ relative to the calibrated value for the relevant model. The vertical axis is the percentage change in the variable of interest.

The results of this exercise are presented in Figure 6. The first thing to note is that accounting for the dynamics of firms does impact the aggregate effects of trade. Within the range of iceberg costs that are considered, the increase in GDP from a given decrease in iceberg costs is approximately doubled when firm dynamics are accounted. For an increase in iceberg costs the difference is also large, with the decline in GDP only being about half as large. So accounting for firms dynamics does matter for aggregates.

The second feature to note is that there is asymmetry in the effect of firm dynamics. For a decrease in iceberg costs, the gains in GDP are *amplified* when firm dynamics are accounted for, where when iceberg costs increase the losses are *dampened*. This is not simply a case of firm dynamics making the economy more or less sensitive to changes in trade costs. The sign of the change in sensitivity actually depends on the level of openness of the economy.

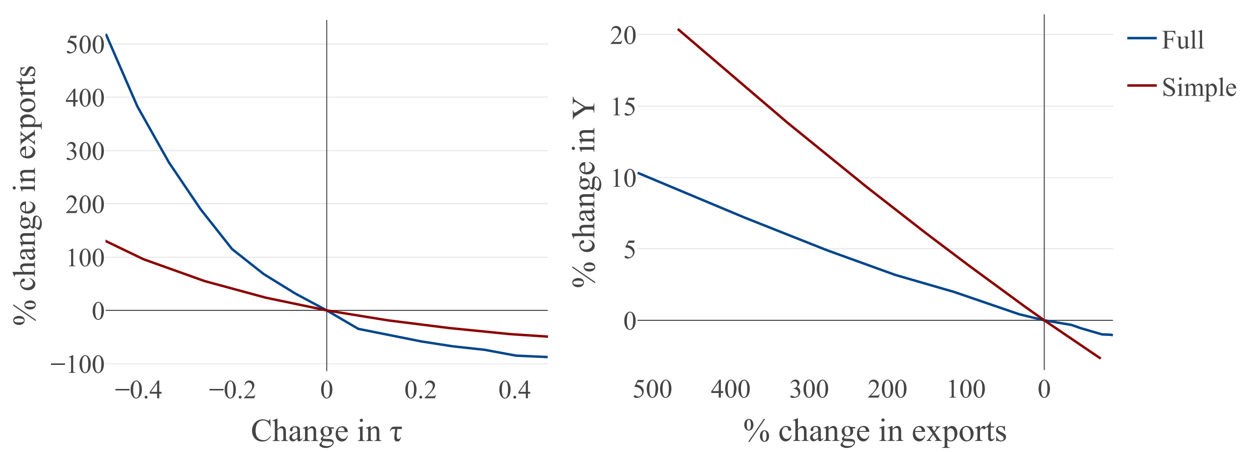
[15]For the full model this corresponds to varying τ from 0.18 to 1.07 and for the simple model the range is from 0.66 to 1.56.



(a) Perfect information model

(b) Exogenous customer accumulation model

Figure 7: Effect of iceberg cost changes under perfect information and exogenous customer accumulation. This figure shows the percentage change in Y as a function of the change in τ for the two special cases of the full model. Panel (a) presents results for the perfect information model in which firms know their demand distributions in both markets at birth. Panel (b) presents results for the case in which all firms acquire the same exogenous number of new customers each period. All changes are relative to the baseline calibration for the relevant model. Results for the full and simple model from Figure 6 are repeated for comparison.



(a) Iceberg costs and exports

(b) Exports and GDP

Figure 8: Decomposition of effect of iceberg costs on GDP through exports. Panel (a) presents the percentage change in aggregate exports as a function of the change in τ for the simple and full models. Panel (b) plots the percentage change in exports and the percentage change in GDP for the same exercise. All changes are relative to the baseline calibration for the relevant model.

To understand these results we perform a number of experiments. The first set of experiments is designed to determine the importance of learning about demand and customer accumulation for the results. To assess the relevance of learning about demand we return to the perfect information model that was introduced in Section 4. Recall that in this model all firms know there demand distributions in Home and Foreign at birth. Figure 7a contains the results for this model, and maintains the lines for the Full and Simple models for reference. The figure clearly shows that despite learning being important for understanding some of the micro details of exporter dynamics, it does not matter for the aggregate effects of changes in trade costs. The change in GDP results from a change in iceberg costs is virtually identical in the perfect information and the full model. We see this as a valuable result because it clearly shows that for the purposes of at least some aggregates, there are moments of the micro data that one can safely ignore when modeling firms. Some intuition for this result is that the information friction in the model primarily affects young exporters. After some years firms accumulate good information about their demand distribution, and their level of expected demand stabilizes (recall Figure 2). Since it is older exporters that account for the majority of exporters, this information friction does not affect the level of trade much and, consequently, does not affect the economy's aggregate response to changes in trade costs.

Having narrowed down our focus for the different between the full and simple models to customer capital, we next evaluate how the endogeneity of the customer choice matters. Specifically, in the full model firms choose the number of customers that they want to acquire in the export market each period, conditional on exporting. To evaluate the relevance of this decision we consider an alternative version of the model in which firms accumulate customers at an exogenously given rate. We pick this rate so that the average growth rate of firms is unchanged and call this model the *exogneous customer accumulation model.* The results for it are presented in Figure 7b, with the the results for the full and simple models again reproduced for comparison. This experiment shows that endogenizing the customer capital choice has an amplification effect. When firms are allowed to choose how many customers they want to acquire the gains from lowering trade costs are amplified, and the losses from increasing these costs are also amplified. Another point to note is that the exogenous customer accumulation model qualitatively produces the same feature as the full model, that gains from lowering iceberg costs are larger than in the simple model, while the losses from increasing these costs are smaller.

The question that remains to be addressed is why having customer accumulation in the model changes the effects of changes in iceberg costs in the way that it does? To assess this we decompose the effects of changes in iceberg costs into (1) the effect on the level of exports and (2) the change in GDP conditional on exports. This analysis is presented in Figure 8 for the full and simple models. Starting with panel (b), the results show that both models have an effectively constant elasticity of output with respect to exports (the lines are linear). This elasticity is smaller in the full model so that, for a given changes in exports, the gains or losses in terms GDP are smaller in the full model. The second piece of the analysis in panel (a) shows that the change in exports for a given change in iceberg costs is larger in the full than the simple model, and the difference between the models is much larger for decreases in iceberg costs. This is clearly the source of of the more convex relationship between iceberg costs and GDP in the full model. Putting the two pieces of the analysis together, the full model has a larger increase in GDP for decreases in iceberg costs because exports increase a lot more. This effect is sufficiently large that it more than offsets the fact that the increase in GPD conditional on an increase in exports is smaller in the full model than the simple model. For an increase in iceberg costs, exports decrease more in the full than the simple model, but this effect is outweighed by the GDP losses being smaller for a given decrease in exports. Overall the losses from higher iceberg costs are smaller in the full model.

6 CONCLUSION

How do the dynamics of exporters alter our understanding of the aggregate effects of trade liberalization and protectionism? To address this question we have developed a general equilibrium trade model with endogenous exporter dynamics that are consistent with the data. These dynamics are driven by a costly process for acquiring customers and imperfect information about demand that can be resolved through learning. By comparing the effects of changes in variable trade costs in our model and a simpler model in which the rich exporter dynamics are omitted, we have shown that these dynamics impact the effects of changes in trade costs. These dynamics make the economy's responsiveness to these changes much more dependent on the initial level of openness: more open economies gain more from lower trade costs than less open ones. Quantitatively the differences can be large. For the US economy the dynamics of firms can amplify the gains from lower trade costs by as much as a factor of two.

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A SIMPLIFIED MODEL

In this section we provide additional details of the simplified model that is introduced in Section 3.2. This model makes two changes to the full model presented in Section 2. First, firms do not face demand shocks so $\xi_{it}(\omega) = 1 \; \forall \, \omega, i, t$. Second, firms don't have to invest in order to make customers aware of their product. Instead we revert to the usual assumption that all agents know about all goods in the economy.

The implications of these changes for key equations are as follows. The consumption of consumer i is now:

$$C_{it} = \left(\int_{\Omega_t} q_{it}(\omega)^{\rho} \right)^{1/\rho}.$$

and the Home price index is

$$P_t := \left(\int_{\Omega_t} p_t(\omega)^{\frac{\rho}{\rho-1}} d\omega \right)^{\frac{\rho-1}{\rho}}.$$

The key differences to the analogous equations for the full model are that the integrals are now over all goods that are sold in Home and there are no demand shocks. The demand function for a firm can be written as

$$q_t(\omega) = \sum_{i \in \mathcal{I}_{\omega t}} q_{it}(\omega) = \frac{Y_t}{P_t} \left(\frac{P_t}{p_t(\omega)} \right)^{\frac{1}{1-\rho}}$$

and the labor demand and profit equations for the Home sales of a Home firm simplify to:

$$l_{Ht}(\omega) = \frac{Y_t}{P_t} \left(\frac{\rho P_t}{w_t} \right)^{\frac{1}{1-\rho}} z_t(\omega)^{\frac{\rho}{1-\rho}}, \quad \pi_{Ht}(\omega) = (1-\rho)\, Y_t \left(\frac{\rho z_t(\omega) P_t}{w_t} \right)^{\frac{\rho}{1-\rho}}.$$

The pricing function is the same as in the full model. The labor demand and profit functions for exporting change analogously and the pricing function for exporting is unchanged.

The export decision in this version of the model simplifies to a static problem. This is because there is no sunk cost of exporting in the model (only a fixed cost) and conditional on exporting the decision of a firm is what price to set, which is a static problem. A firm will choose to export in period t if its profit from exporting in that period is positive:

$$(1-\rho)\, Y_t^* \left(\frac{\rho z_t(\omega) P_t^*}{w_t(1+\tau)} \right)^{\frac{\rho}{1-\rho}} - \gamma_x > 0.$$

The free entry condition is:

$$\mathbb{E}[V(z)] = \gamma_e w.$$

where $V(z)$, the value of a firm with productivity z is:

$$V(z) = \pi_H(z) + x[\pi_F(z) - \gamma_x] + (1-\delta)\theta \mathbb{E}[V(z')|z].$$

The equation for aggregate income is:

$$Y_t = w_t + \int \Big(\pi_{Ht}(z) + x(z)[\pi_{Ft}(z) - \gamma_x w_t] \Big) f_t(z) dz - M_{et} \gamma_e w_t$$

and the labor market clearing condition is:

$$M_{et}\gamma_e + \int \Big(l_{Ht}(z) + x(z)[l_{Ft}(z) + \gamma_x]\Big) f_t(z)dz = 1$$

The market clearing conditions for varieties sold in Home and Foreign are the same as in equations (21) and (22).

Since the only state variable of a firm in this version of the model is z the distribution of firms over states is much simpler than in the full model. Let Σ_Z be the set of all subsets of Z and let $\mathcal{Z}$ denote a single subset. Let $Q : Z \times \Sigma_Z \to [0, 1]$ be the transition function for the distribution of incumbent firms, so $Q(z, \mathcal{Z})$ is the probability that a firm with productivity z transitions to a value of $z' \in \mathcal{Z}$ next period. This transition function is:

$$Q(z, \mathcal{Z}) = (1 - \delta) \sum_{z' \in \mathcal{Z}} G(z'|z).$$

The distribution of entrants is:

$$Q_e(\mathcal{Z}) = \sum_{z \in \mathcal{Z}} G(z).$$

The distribution of firms over Z therefore evolves according to

$$F_t(\mathcal{Z}) = \frac{M_{H,t-1} \sum_{z \in Z} Q(z, \mathcal{Z}) + M_{e,t} Q_e(\mathcal{Z})}{(1 - \delta) M_{H,t-1} + M_{e,t}}$$

and a stationary distribution is

$$F(\mathcal{Z}) = \sum_{z \in Z} Q(z, \mathcal{Z}) + \frac{M_e}{M_H} Q_e(\mathcal{Z}).$$

B COMPUTATIONAL DETAILS

In this section we outline the core algorithm for solving the full model.[16] One price can be normalized and we normalize the aggregate price index (P) to be equal to 1. The main algorithm is composed of five steps:

1. *Guess aggregates*: Guess values for the wage (w), aggregate income (Y), and the mass of Home firms (M_H).

2. *Setup*: The firm's labor demand problem is a static function of the individual firm states (N, α, β, z). We write routines to compute labor demand (equations 8 and 11), domestic profits (equation 9), and the expected value of foreign profits (expectation of equation 12, conditional on α and β) at any state.

3. *Export decision*: Taking aggregates w and Y as given, we solve the firm's export problem on a grid over (N, α, β, z). To do this we use value function iteration over equations (15) and (16) to compute the continuation value of exporting and not-exporting. The output of this step is a policy rule mapping

[16] Our routines are implemented efficiently in the Julia language. Source code is available upon request

(N, α, β, z) into the optimal number of domestic customers to acquire and another policy mapping $(N^*, \alpha^*, \beta^*, z)$ into an export decision and the optimal number of foreign customers to acquire. These problems are solved independently as they do not interact with one another.

4. *Stationary Distribution*: Given the aggregates and the firms' customer acquisition decisions, we solve for an approximation of the stationary distribution of firms state variables, which we denote as

$$f(N, \alpha, \beta, z, \lambda, N^*, \alpha^*, \beta^*, \lambda^*).$$

5. *Update aggregates*: Given an approximation of the stationary distribution we check that the labor market clearing condition (20), the definition of aggregate income (18) and the free entry condition (19) hold. If they don't we update our guesses for w, Y and M_H and repeat the above steps.